Marek Bienias

Some properties of compact preserving functions

Joint results with Taras Banakh, Artur Bartoszewicz and Szymon Głąb

Notion

Definition

A function $f : X \to Y$ between topological spaces is called compact-preserving, provided the set $f(K) \subseteq Y$ is compact for any compact set $K \subseteq X$

Example

- any continuous function;
- any function with finite range.

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A function $f : \mathbb{R} \to \mathbb{R}$ is continuous iff f is compact preserving and has the Darboux property (i.e. maps connected sets to connected sets).

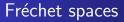
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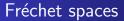
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We recall that a topological space X is

- Fréchet if for each A ⊂ X and a ∈ Ā there is {a_n}_{n∈ω} ⊂ A that converges to a;
- strong Fréchet if for any decreasing sequence {A_n}_{n∈ω} ⊆ X and any a ∈ ∩_{n∈ω} Ā_n there is a sequence a_n ∈ A_n, n ∈ ω, that converges to a.



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Let $f: X \to Y$ and $x \in X$.

Definition

$$f[x] = \{y \in Y : x \in cl_X(f^{-1}(y))\}$$

 $= \bigcap \{ f(O_x) : O_x \text{ is a neighborhood of } x \text{ in } X \},\$

f[x] can be interpreted as the oscillation of f at x. If f is continuous at x and Y is a T_1 -space, then the set $f[x] = \{f(x)\}.$

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Theorem 1

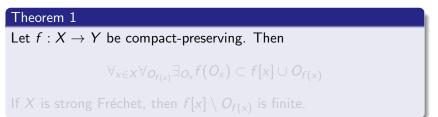
Let $f : X \to Y$ be compact-preserving. Then

$\forall_{x \in X} \forall_{O_{f(x)}} \exists_{O_x} f(O_x) \subset f[x] \cup O_{f(x)}$

If X is strong Fréchet, then $f[x] \setminus O_{f(x)}$ is finite.

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- Step 2 Denote $A = f^{-1}(Y \setminus (f[x_0] \cup O_{f(x_0)}))$ and notice than $x_0 \in \overline{A}$;
- Step 3 Take a sequence $\{x_n\} \subseteq A$ s.t. $x_n \rightarrow x_0$;
- Step 4 Observe that $\{f(x_n)\}$ is one-to-one from some place;
- Step 5 Notice that a set $K = \{f(x_0)\} \cup \{f(x_n)\}_{n \in \omega}$ is compact and infinite;
- Step 6 Throw out a non-isolated point y_0 from K, $K \setminus \{y_0\}$ is not compact;
- Step 7 Observe that $y_0 \neq f(x_0)$;

Step 8 But $K \setminus \{y_0\} = f(S)$ where $S = \{x_0\} \cup \{x_n\}_{n \in \omega} \setminus f^{-1}(y_0)$ is compact.

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Corollary

Function $f : X \rightarrow Y$ is compact-preserving if (and only if)

$$\forall_{x \in X} \exists_{K_x \subseteq Y} \forall_{O_{f(x)}} \exists_{O_x} f(O_x) \subset K_x \cup O_{f(x)}$$

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Let $f: X \to Y$. A sequence $\{x_n\}_{n \in \omega} \subseteq X$ is called

- *injective* if $x_n \neq x_m$ for $n \neq m$;
- *f*-injective if $f(x_n) \neq f(x_m)$ for $n \neq m$.

Observation

For any compact-preserving function $f : X \to Y$ from a topological space X to a Hausdorff space Y and each f-injective sequence $\{x_n\}_{n\in\omega} \subset X$ that converges to a point $x \in X$ the sequence $\{f(x_n)\}_{n\in\omega}$ converges to the point f(x).

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 O_x ⊂ X of x, the image f(O_x) is finite;
- *locally infinite* at $x \in X$ if f is not locally finite at x;

Corollary

X- sequentially Hausdorff Fréchet space, Y- Hausdorff space If f is compact-preserving and locally infinite at each point $x \in X$ then $f : X \to Y$ is continuous.

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- Let L be a vector space and a set A ⊆ L. We say that A is κ-lineable if A ∪ {0} contains a κ-dimensional vector space;
- ② Let L be a Banach space and a set A ⊆ L. We say that A is spaceable if A ∪ {0} contains an infinite dimensional closed vector space;
- One Let L be a linear commutative algebra and a set A ⊆ L. We say that A is κ-algebrable if A ∪ {0} contains a κ-generated algebra B (i.e. the minimal system of generators of B has cardinality κ).

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Let \mathcal{L} be a linear commutative algebra and a set $A \subseteq \mathcal{L}$. We say that A is strongly κ -algebrable if $A \cup \{0\}$ contains a κ -generated algebra B that is isomorphic with a free algebra.

3.1

 $\mathcal{EDC}(\mathbb{R})$ is the set of all nowhere continuous compact-to-compact functions.

Theorem

The set $\mathcal{EDF}(\mathbb{R})$ is 2^c-algebrable but it is not strongly 1-algebrable.

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 $\mathcal{EDC}(\mathbb{R})$ is the set of all nowhere continuous compact-to-compact functions.

Theorem

The set $\mathcal{EDF}(\mathbb{R})$ is 2^c-algebrable but it is not strongly 1-algebrable.

Corollary

The set $\mathcal{EDC}(\mathbb{R})$ is 2^c-algebrable.

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Question 2, Hejnice 2012

Is there a function $f \in \mathcal{EDC}(\mathbb{R})$ that has infinitely many values on each interval?

Corollary

X- sequentially Hausdorff Fréchet space, Y- Hausdorff space If f is compact-preserving and locally infinite at each point $x \in X$ then $f : X \to Y$ is continuous.

Answer

No.

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Answer

No.

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Thank you for your attention :)

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